

# Throughput Optimal Scheduling in Multi-hop Wireless Networks

Kyomin Jung (KAIST)

joint work with

Ramakrishna Gummadi (UIUC)

Devavrat Shah (MIT)

RS Sreenivas (UIUC)

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# Network Capacity: Basic Problem

- Model of Wireless Network
  1. Directed Graph
  2. Protocol Model (i.e. no Coding in network, or any Information theoretic schemes, no exploiting of broadcast multipath, etc.)
  3. Interference constraints: Specified via link pair conflicts (Ex: k-hop, node exclusive, etc.)

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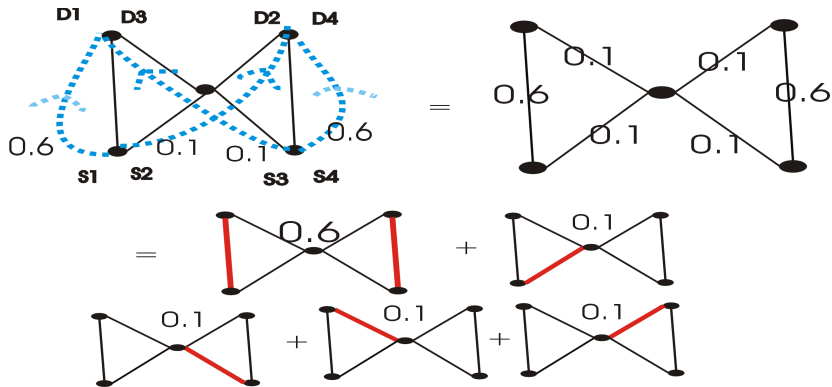
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- Given: (i)  $m$  S-D pairs; (ii) an  $m$ -dim rate vector,  $\mathbf{r}$

Question: Does  $\mathbf{r} \in \mathcal{F}$ ? (i.e., can we satisfy the following two constraints simultaneously?)

1.  $m$  flows -  $i^{\text{th}}$  flow of value  $r_i$  between S-D pair  $i$  (routing)
2. Sum of flows convex combination of non-conflicting link subsets. (scheduling)  
(i.e., is there a TDMA scheme which supports the sum of flows?)

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We say end to end rate vector,  $(0.6, 0.1, 0.1, 0.6)$  is feasible

## Background on rate feasibility

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- Given end-to-end rates and a flow routing decomposition, end-to-end feasibility reduces to **link rate feasibility**
- Our previous work on **single hop**: “*Feasible Rate Allocation in Wireless Networks*”, *INFOCOM '08* provides approximate poly time oracles for link rate feasibility in restricted graphs.

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- **Our Result**: Polynomial algorithm for  $n^2$  dimensional unicast capacity upto arbitrary accuracy when network graph allows for MWIS approximation (in many practical situations, it does)

*Algorithm declares in poly time:*

*'YES', if  $(1 + 2\epsilon)\mathbf{r} \in \mathcal{F}$*

*'NO' if  $(1 - 2\epsilon)\mathbf{r} \notin \mathcal{F}$*

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Recall: Markov Chain for i.i.d. packet arrivals stable under “MWIS routing/scheduling” iff  $r \in \mathcal{F}$ ; “*Stability properties...*”, Tassiulas, Ephremides, TAC '92

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- **Caveat:** Stability is only existential statement. But, how long do we need to observe queue lengths and how do we make the call on feasibility? (in poly time)

# Challenges

- **Key issues** to be addressed:
  - (i) Does *approximate* MWIS  $\Rightarrow$  *approximate* stability?
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- **Deterministic real valued queue length process  $\Rightarrow$  explicit bounds**

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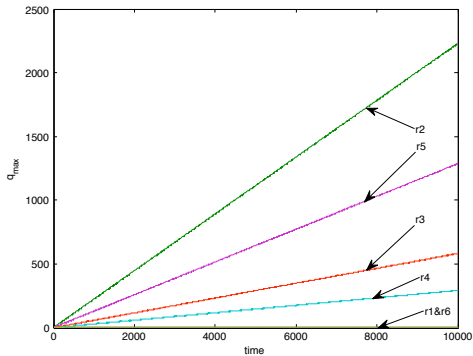
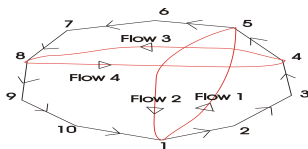
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- $t$  time slots of  $\epsilon/2$ -MWIS  $\Rightarrow$  Feasibility of  $(1 \pm \epsilon(t))\mathbf{r}$  where  $\epsilon(t) = 2 \min \left( \sqrt{\frac{q^{\max}(t)p_2(n)}{t}}, \frac{p_1(n)}{q^{\max}(t)} \right)$  when  $\epsilon < \epsilon(t) < 1/2$   
*Assured that  $\epsilon(t)$  decreases below any  $\epsilon > 0$  in poly time.*

# Comments

- $p_1(n) = n^{7.5}$ ,  $p_2(n) = n^2$ , but the algorithm could be potentially more efficient in practice than the guarantees provided.



# The approximability of MWIS

- Is this too strong an assumption for practical use?

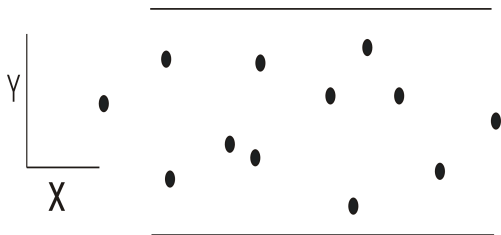
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- Graphs with polynomial growth:
  1. Geometric random graph with  $O(\log n)$  communication radius
  2. Arbitrary geographic graphs with bounded density and communication radius

## A Special case with Exact algorithm



- Motivated by IVHS applications.
- Bounded radius of communication and interference.
- End-to-end rate feasibility can be posed as a polynomial LP by extending our previous work on link feasibility based on a fractional coloring algorithm.

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- **Existing literature** considers the problem of understanding the capacity for an ensemble of networks with a probabilistic distribution and as the network size goes to infinity (**scaling laws**).
- A byproduct of our work: **Transient analysis** of the approximate max weight scheduling algorithm for deterministic arrivals.