Opportunistic Scheduling in Wireless Networks

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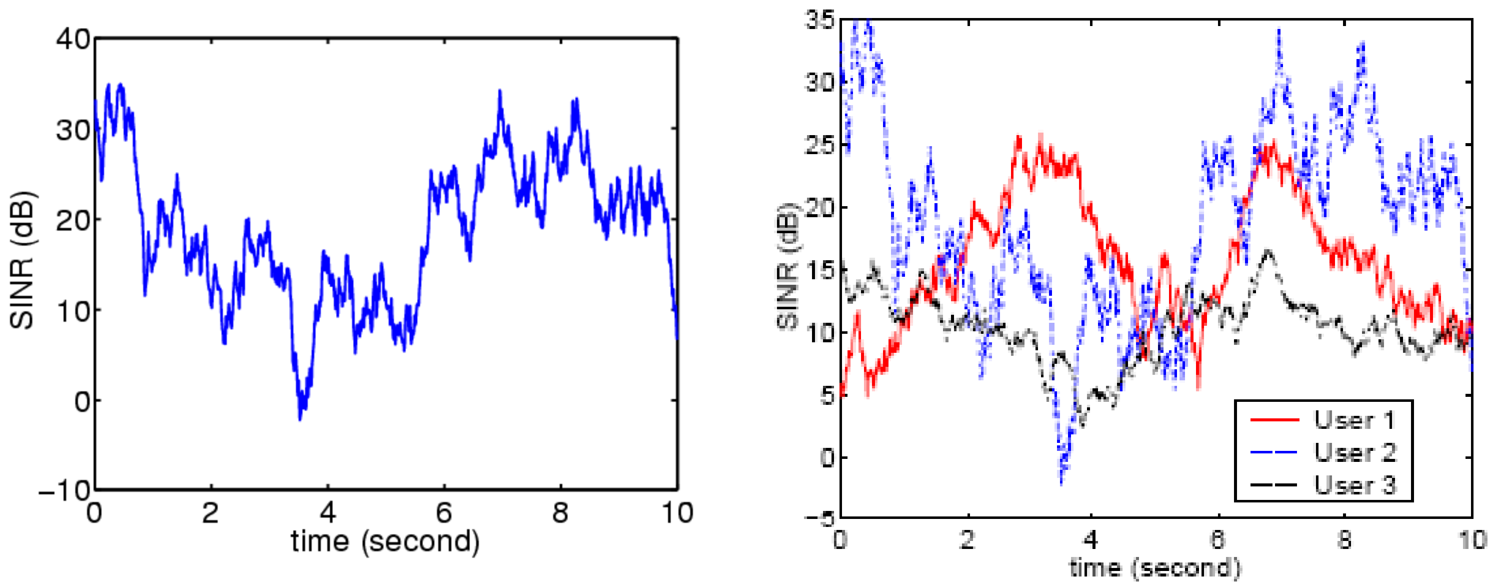
Time-varying channel conditions

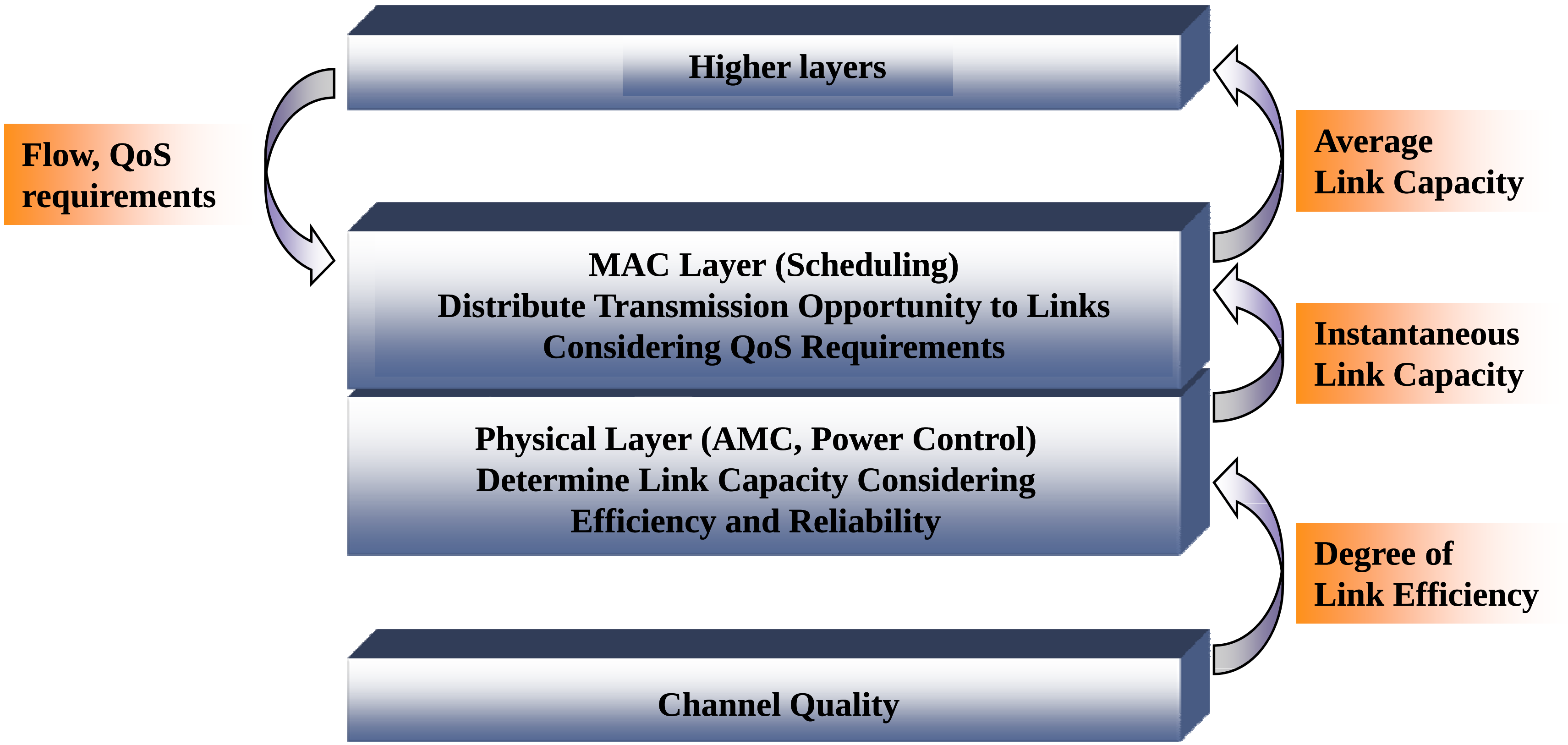
- Propagation environment
  - Path loss
  - Shadowing or slow-fading: Log-normal distribution
  - Fast-fading or multipath fading: Rayleigh or Ricean distribution
- Both received signal and interference are time varying
- A measure of channel quality: SINR (Signal to Interference plus Noise Ratio)

$$\text{SINR} = \frac{\text{desired signal power}}{\text{interference power} + \text{background noise power}}$$

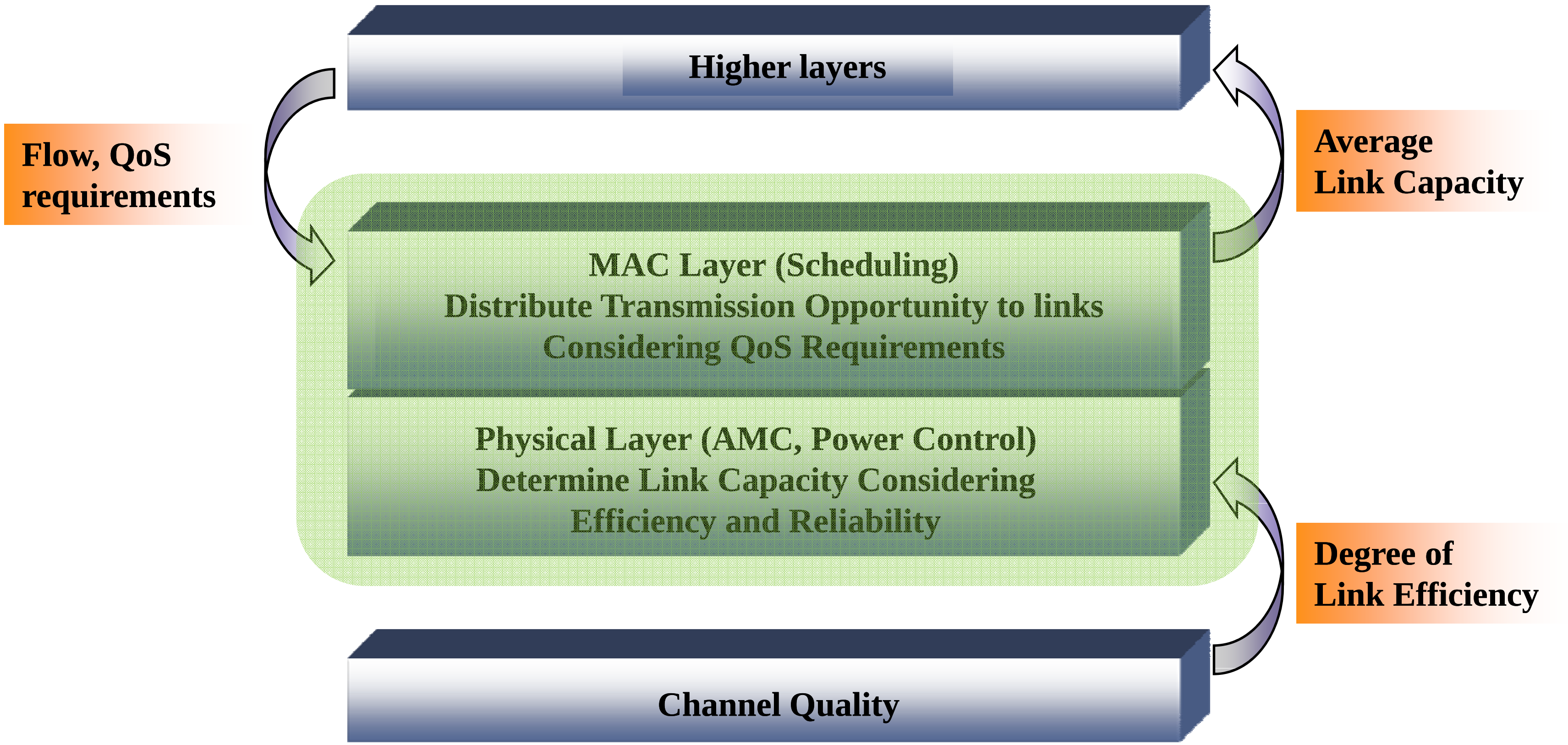
Time-varying channel conditions



MAC-PHY Cross-Layer Optimization



Opportunistic Scheduling



What is opportunistic scheduling?

- Trade-off between efficiency and fairness due to
  - multi-class users
  - time-varying and location-dependent channel condition of each user
  - good efficiency-fairness trade-off can be achieved by an appropriate RRM and scheduling
- Opportunistic scheduling
  - scheduling and resource allocation exploiting time-varying channel conditions of each user to
    - maximize ***system performance***
    - while satisfying ***QoS or fairness*** requirements of each user

Utility function

- A function of resource allocation
- Can be used as
  - Measure of QoS, satisfaction, or performance for service as a function of resource allocation
  - Knops to control efficiency-fairness tradeoff

Utility function

- Instantaneous rate based utility function
  - Defined as a function of instantaneous data rate at time t
  - Instantaneous utility at time t: $U(x(t))$
    - $x(t)$ is data rate at time t
  - Average utility: $E_t[U(x(t))]$

- Average rate based utility function
  - Defined as a function of average data rate
  - Average data rate: $x = E_t[x(t)]$
  - Utility: $U(x)=U(E_t[x(t)])$

Fairness

- Threshold based fairness
  - Each user has its explicit fairness requirement
  - Examples
    - $E_t[U(x(t))] \geq U_{\min}$
    - $U(E_t[x(t)]) \geq U_{\min}$
- Utility based fairness
  - Desired fairness can be achieved by using appropriate utility functions
  - Examples

$$U^\alpha(x) = \begin{cases} (1-\alpha)^{-1}x^{1-\alpha}, & \text{if } \alpha \neq 1 \\ \log x, & \text{otherwise} \end{cases}$$

    - if α=1: proportional fairness
    - if α=∞: max-min fairness
    - In general, as α increases efficiency decreases and fairness increases

Two famous frameworks for opportunistic scheduling

- Liu's framework
  - Instantaneous rate based utility function
  - Threshold based fairness
  - Parameter based algorithm

- Agrawal's framework
  - Average rate based utility function
  - Utility based fairness
  - Gradient based algorithm

X. Liu, E. K. P. Chong, and N. B. Shroff. A framework for opportunistic scheduling in wireless networks. *Computer Networks*, 41(4):451–474, Mar. 2003.

R. Agrawal and V. Subramanian. Optimality of certain channel aware scheduling policies. In *Allerton Conference on Communication, Control and Computing*, 2002.

Liu’s Framework

X. Liu, E. K. P. Chong, and N. B. Shroff. A framework for opportunistic scheduling in wireless networks. *Computer Networks*, 41(4):451–474, Mar. 2003.

System model

- Set ***M*** of users in a single cell
- Time-slotted system with fixed transmit power
  - Time-slot is resource shared by all users
- A time-slot is exclusively used by one user within a cell
  - TDMA
- U_i: utility function of user i
  - ***Function of instantaneous data rate***
- x_i: achievable data rate of user i in a time-slot
  - Random variable depending on channel condition
- Each user has an ***explicit fairness requirement***
- Θ: set of all scheduling policies
- Q: scheduling policy

$$Q(\boldsymbol{U}(\boldsymbol{x})) = i$$

  - Given $\boldsymbol{U}(\boldsymbol{x}) = [U_1(x_1), \ldots, U_M(x_M)]$, schedule user i

Scheduling problem

$$\text{maximize}_{Q \in \Theta} \quad \sum_{i \in M} E\left[U_i(x_i) 1_{\{Q(\boldsymbol{U})=i\}}\right]$$

$$\text{subject to} \quad E\left[U_i(x_i) 1_{\{Q(\boldsymbol{U}(\boldsymbol{x}))=i\}}\right] \geq C_i, \forall i$$

- Maximizing the sum of expected utilities of all users (network utility)
  - while satisfying the minimum expected utility requirement of each user

Scheduling algorithm

$$Q(\boldsymbol{U}(\boldsymbol{x})) = \arg\max_{i \in M} \left[\alpha_i^* U_i(x_i)\right]$$

Where α_i^*'s are parameters that satisfy

- $\min_i (\alpha_i^*) = 1$
- For all i, $E\left[U_i(x_i) 1_{\{Q(\boldsymbol{U}(\boldsymbol{x}))=i\}}\right] \geq C_i$
- For all i, if $E\left[U_i(x_i) 1_{\{Q(\boldsymbol{U}(\boldsymbol{x}))=i\}}\right] > C_i$, then $\alpha_i^* = 1$

Such optimal α_i^*'s can be obtained by using stochastic approximation

Agrawal's Framework

R. Agrawal and V. Subramanian. Optimality of certain channel aware scheduling policies. In *Allerton Conference on Communication, Control and Computing*, 2002.

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  - TDMA
- U_i: utility function of user i
  - Function of ***average data rate***
- x_i: achievable data rate of user i in a time-slot
  - Random variable depending on channel condition
  - Each user has ***no explicit fairness requirement***
- Θ: set of all scheduling policies
- Q: scheduling policy

$$Q(\boldsymbol{U}(\boldsymbol{x})) = i$$

  - Given $\boldsymbol{U}(\boldsymbol{x}) = [U_1(x_1), \ldots, U_M(x_M)]$, schedule user i

Scheduling problem

$$\text{maximize}_{\mathrm{Q}\in\Theta} \quad \sum_{i\in M}\left[U_i(E[x_i 1_{\{Q(\boldsymbol{U})=i\}}])\right]$$

- Maximizing the sum of utilities of all users (network utility)
- No explicit fairness requirement
- Fairness can be achieved by using appropriate utility functions

Scheduling algorithm

$$Q(\boldsymbol{U}(\boldsymbol{x})) = \underset{i \in M}{\operatorname{argmax}}(\nabla U_i(w_i)x_i)$$

Where w_i's are achieved average data rate so far

- Example: Proportional fair scheduling
  - $U_i(x_i) = \log(x_i)$

$$Q(\boldsymbol{U}(\boldsymbol{x})) = \underset{i \in M}{\operatorname{argmax}}(\nabla U_i(w_i)x_i)$$

$$= \underset{i \in M}{\operatorname{argmax}}\left(\frac{x_i}{w_i}\right)$$

Reinterpretation of Liu and Agrawal's frameworks: unified dual framework

J.-A. Kwon, B.-G. Kim and J.-W. Lee, " A unified framework for opportunistic fair scheduling in wireless networks: a dual approach", submitted

System model

- Same as before, except
  - Scheduling scheme decides which user to assign to a time-slot randomly
  - User i is selected with probability $p_i(\boldsymbol{x})$

$$\sum_{i \in M} p_i(\boldsymbol{x}) \leq 1$$

$$0 \leq p_i(\boldsymbol{x}) \leq 1, \forall i$$

Liu's problem and dual approach

$$\begin{aligned} \text{maximize}_{\boldsymbol{p}} \quad & \sum_{i \in M} E\left[U_i(x_i) p_i(\boldsymbol{x})\right] \\ \text{subject to} \quad & E\left[U_i(x_i) p_i(\boldsymbol{x})\right] \geq C_i, \forall i \\ & \sum_{i \in M} p_i(\boldsymbol{x}) \leq 1 \\ & 0 \leq p_i(\boldsymbol{x}) \leq 1, \forall i \end{aligned}$$

- Lagrangian function

$$L(\boldsymbol{p}, \boldsymbol{\mu}) = \sum_{i \in M} E\left[U_i(x_i) p_i(\boldsymbol{x})\right] + \sum_{i \in M} \mu_i \left(C_i - E\left[U_i(x_i) p_i(\boldsymbol{x})\right]\right)$$

- Dual objective function

$$D(\boldsymbol{\mu}) = \max_{\substack{\sum p_i(\boldsymbol{x}) \leq 1 \\ 0 \leq p_i(\boldsymbol{x}) \leq 1, \forall i}} L(\boldsymbol{p}, \boldsymbol{\mu})$$

- Dual problem

$$\begin{aligned} \text{minimize} \quad & D(\boldsymbol{\mu}) \\ \text{subject to} \quad & \boldsymbol{\mu} \geq \boldsymbol{0} \end{aligned}$$

Dual based scheduling

- Dual based scheduling

$$p_i^*(\boldsymbol{x}) = \begin{cases} 1, & \text{if } i = \arg\max \{(1+\mu_i^*) U_i(x_i)\} \\ 0, & \text{otherwise} \end{cases}$$

- μ_i^*'s: dual variables that satisfy
  - Non-negativity condition

    $$\mu_i^* \geq 0$$

  - Feasibility condition

    $$E[U_i(x_i) p_i^*(\boldsymbol{x})] \geq C_i, \forall i$$

  - Complementary slackness condition

    $$\mu_i^* \left(C_i - E[U_i(x_i) p_i^*(\boldsymbol{x})]\right) = 0, \forall i$$

- Liu's scheduling

$$Q(\boldsymbol{U}(\boldsymbol{x})) = \underset{i \in M}{\arg\max}(\alpha_i^* U_i(x_i))$$

- α_i^*'s satisfy
  - $\min_i (\alpha_i^*) = 1$
  - For all i, $E[U_i(x_i) 1_{\{Q(\boldsymbol{U}(\boldsymbol{x}))=i\}}] \geq C_i$
  - For all i, if $E[U_i(x_i) 1_{\{Q(\boldsymbol{U}(\boldsymbol{x}))=i\}}] > C_i$, then $\alpha_i^* = 1$

$$\alpha_i^* = 1 + \mu_i^*$$

Agrawal's problem and dual approach

$$\begin{aligned} \text{maximize}_{\boldsymbol{p}} \quad & \sum_{i \in M} U_i(E[x_i p_i(\boldsymbol{x})]) \\ \text{subject to} \quad & \sum_{i \in M} p_i(\boldsymbol{x}) \le 1 \\ & 0 \le p_i(\boldsymbol{x}) \le 1, \forall i \end{aligned}$$

$\Rightarrow$

$$\begin{aligned} \text{maximize}_{\boldsymbol{p},\boldsymbol{w}} \quad & \sum_{i \in M} U_i(w_i) \\ \text{subject to} \quad & w_i \le E[x_i p_i(\boldsymbol{x})] \\ & \sum_{i \in M} p_i(\boldsymbol{x}) \le 1 \\ & 0 \le p_i(\boldsymbol{x}) \le 1, \forall i \end{aligned}$$

- Lagrangian function

$$L(\boldsymbol{p},\boldsymbol{\mu}) = \sum_{i \in M} U_i(w_i) + \sum_{i \in M} \mu_i \left(E[x_i p_i(\boldsymbol{x})] - w_i \right)$$

- Dual objective function

$$D(\boldsymbol{\mu}) = \max_{\substack{\sum p_i(\boldsymbol{x}) \le 1 \\ 0 \le p_i(\boldsymbol{x}) \le 1, \forall i}} L(\boldsymbol{p},\boldsymbol{\mu})$$

- Dual problem

$$\begin{aligned} \text{minimize} \quad & D(\boldsymbol{\mu}) \\ \text{subject to} \quad & \boldsymbol{\mu} \ge \boldsymbol{0} \end{aligned}$$

Dual based scheduling

- Dual based scheduling

$$p_i^*(\boldsymbol{x}) = \begin{cases} 1, & \text{if } i = \operatorname{argmax}\{\mu_i^* x_i\} \\ 0, & \text{otherwise} \end{cases}$$

- μ_i^*: optimal dual variables that satisfy

$$\mu_i^* = \nabla U_i(w_i^*)$$

$$p_i^*(\boldsymbol{x}) = \begin{cases} 1, & \text{if } i = \operatorname{argmax}\{\nabla U_i(w_i^*) x_i\} \\ 0, & \text{otherwise} \end{cases}$$

- Agrawal's scheduling

$$Q(\boldsymbol{U}(\boldsymbol{x})) = \operatorname*{argmax}_{i \in M}(\nabla U_i(w_i) x_i)$$

Conclusion

- Opportunistic scheduling is an essential technique in current and future wireless networks to improve system efficiency
  - Proportionally fair scheduling
- Two famous frameworks
  - Liu's framework
  - Agrawal's framework
  - Both frameworks can be reinterpreted in a unified dual framework
- Dual framework can be applied more general problem settings