End-to-End Rate Control in Communication Networks Considering User-Level Satisfactions

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NUM framework

$$\begin{aligned} &\text{maximize} && \textstyle\sum_s U_s(x_s) \\ &\text{subject to} && \textstyle\sum_{s \in S(l)} x_s \leq c_l, \quad \forall l, \\ & && \mathbf{x} \succeq 0. \end{aligned}$$

- U_s: utility function of user *s*
- x_s: data rate of user *s*
- c_l: capacity of link *l*
- $S(l)$: set of sources on link *l*

❖ **Utility function:** $U_s(x_s)$

- Assigned to each user *s*
- A function of allocated resource x_s
- Measure of QoS, satisfaction

❖ **Network Utility:** $\sum_s U_s(x_s)$

- Sum of utilities of all users in the system

❖ **Network Utility Maximization (NUM)**

- Allocate resources such that network utility is maximized

Utility function in communication network

❖ Communication networks

- A user has two-way communication
  - ✓ Transmitting and receiving sessions for a single communication
  - ✓ User's satisfaction depends on both sessions

❖ Previous resource allocation algorithm…

- Considering only one session (transmitting session)
  - ✓ Utility function is defined as a function of only transmitting data rate
    - ➢ Session-level utility function
  - ✓ Receiving rate is indirectly and independently controlled by correspondent

❖ We propose new concept for utility function of each user

- Jointly considering transmitting and receiving sessions
  - ✓ Utility function is defined as a function of transmitting and receiving data rates
    - ➢ User-level utility function
  - ✓ Receiving rate is directly controlled by the user

System model

- *N*: a set of pairs of users (i_0, i_1)
  - User pair (i_0, i_1): users i_0 and i_1 communicate with each other

- $x_{i_0}^t$: transmitting rate of user i_0
- $x_{i_0}^r$: receiving rate of user i_0

$$\text{Constraint 1: } x_{i_0}^t = x_{i_1}^r \quad \text{and} \quad x_{i_0}^r = x_{i_1}^t$$

- Utility function
  - Session-level utility function

$$u_{i_j}^t(x_{i_j}^t) \text{ and } u_{i_j}^r(x_{i_j}^r)$$

  - User-level utility function

$$U_{i_j}\left(u_{i_j}^t(x_{i_j}^t), u_{i_j}^r(x_{i_j}^r)\right)$$

System model

- L: a set of directional links
- C_l: capacity of link l
- $S^t(l)$: a set of users whose transmitting sessions use link l

$$\text{Constraint 2}: \sum_{i_j \in S^t(l)} x_{i_j}^t \le C_l, \quad \forall l \in L$$

Optimization problem

❖ **Network utility maximization**

$$
\begin{aligned}
\text{maximize} \quad & \sum_{i\in N}\left\{U_{i_0}\left(u_{i_0}^t\left(x_{i_0}^t\right), u_{i_0}^r\left(x_{i_0}^r\right)\right)+U_{i_1}\left(u_{i_1}^t\left(x_{i_1}^t\right), u_{i_1}^r\left(x_{i_1}^r\right)\right)\right\} \\
\text{subject to} \quad & x_{i_0}^t = x_{i_1}^r,\ x_{i_0}^r = x_{i_1}^t, && \forall i \in N, \\
& \sum_{i_j \in S^t(l)} x_{i_j}^t \le C_l, && \forall l \in L, \\
& x_{i_0}^t \ge 0,\ x_{i_0}^r \ge 0,\ x_{i_1}^t \ge 0,\ x_{i_1}^r \ge 0, && \forall i \in N.
\end{aligned}
$$

❖ **The above problem can be solved by dual method**

Dual method

❖ **Lagrangian function**

$$L(\mathbf{x}_0, \mathbf{x}_1, \boldsymbol{\lambda}, \boldsymbol{\mu}_0, \boldsymbol{\mu}_1) = \sum_{i \in N} U_{i0}\left(u^t_{i_0}(x^t_{i_0}), u^r_{i_0}(x^r_{i_0})\right) + U_{i_1}\left(u^t_{i_1}(x^t_{i_1}), u^r_{i_1}(x^r_{i_1})\right)$$

$$+ \sum_{i \in N} \mu_{i_1}\left(x^t_{i_0} - x^r_{i_1}\right) + \sum_{i \in N} \mu_{i0}\left(x^t_{i_1} - x^r_{i_0}\right) + \sum_{l \in L} \lambda_l \left(C_l - \sum_{i_j \in S^t(l)} x^t_{i_j}\right)$$

❖ **Dual objective function**

$$D(\boldsymbol{\lambda}, \boldsymbol{\mu}_0, \boldsymbol{\mu}_1) = \max_{x^t_{i_0} = x^r_{i_1},\, x^r_{i_0} = x^t_{i_1},\, \mathbf{x} \geq \mathbf{0},\, \forall i \in N} L(\mathbf{x_0}, \mathbf{x_1}, \boldsymbol{\lambda}, \boldsymbol{\mu}_0, \boldsymbol{\mu}_1)$$

❖ **Dual problem**

$$\min_{\boldsymbol{\lambda} \geq \mathbf{0}} D(\boldsymbol{\lambda}, \boldsymbol{\mu}_0, \boldsymbol{\mu}_1)$$

Decomposition

❖ **Dual objective function is decomposed for each user *i_j***

$$\begin{aligned} &\text{maximize} \quad U_{i_0}\left(u^t_{i_0}(x^t_{i_0}), u^r_{i_0}(x^r_{i_0})\right) - x^t_{i_0} \textstyle\sum_{l \in L^t(i_0)} \lambda_l + \mu_{i_1} x^t_{i_0} - \mu_{i_0} x^r_{i_0} \\ &\text{subject to} \quad x^t_{i_0} \geq 0, \;\; x^r_{i_0} \geq 0 \end{aligned}$$

and

$$\begin{aligned} &\text{maximize} \quad U_{i_1}\left(u^t_{i_1}(x^t_{i_1}), u^r_{i_1}(x^r_{i_1})\right) - x^t_{i_1} \textstyle\sum_{l \in L^t(i_1)} \lambda_l + \mu_{i_0} x^t_{i_1} - \mu_{i_1} x^r_{i_1} \\ &\text{subject to} \quad x^t_{i_1} \geq 0, \;\; x^r_{i_1} \geq 0. \end{aligned}$$

❖ **The above problems can be easily solved**

- Convex optimization problem with simple constraints

❖ $x^t_{i_j}(\boldsymbol{\lambda}, \boldsymbol{\mu}_0, \boldsymbol{\mu}_1)$ and $x^r_{i_j}(\boldsymbol{\lambda}, \boldsymbol{\mu}_0, \boldsymbol{\mu}_1)$

- Optimal solution of the above problem given $\boldsymbol{\lambda}, \boldsymbol{\mu}_0, \boldsymbol{\mu}_1$

Gradient projection algorithm

- **Dual problem can be solved by using the gradient projection algorithm**

$$\lambda_l^{(n+1)} = \left[\lambda_l^{(n)} - a^{(n)} \left(C_l - \sum_{i \in S^t(l)} x_{i_j}^t(\boldsymbol{\lambda}^{(n)}, \boldsymbol{\mu}_0^{(n)}, \boldsymbol{\mu}_1^{(n)}) \right)\right]^+, \forall l \in L$$

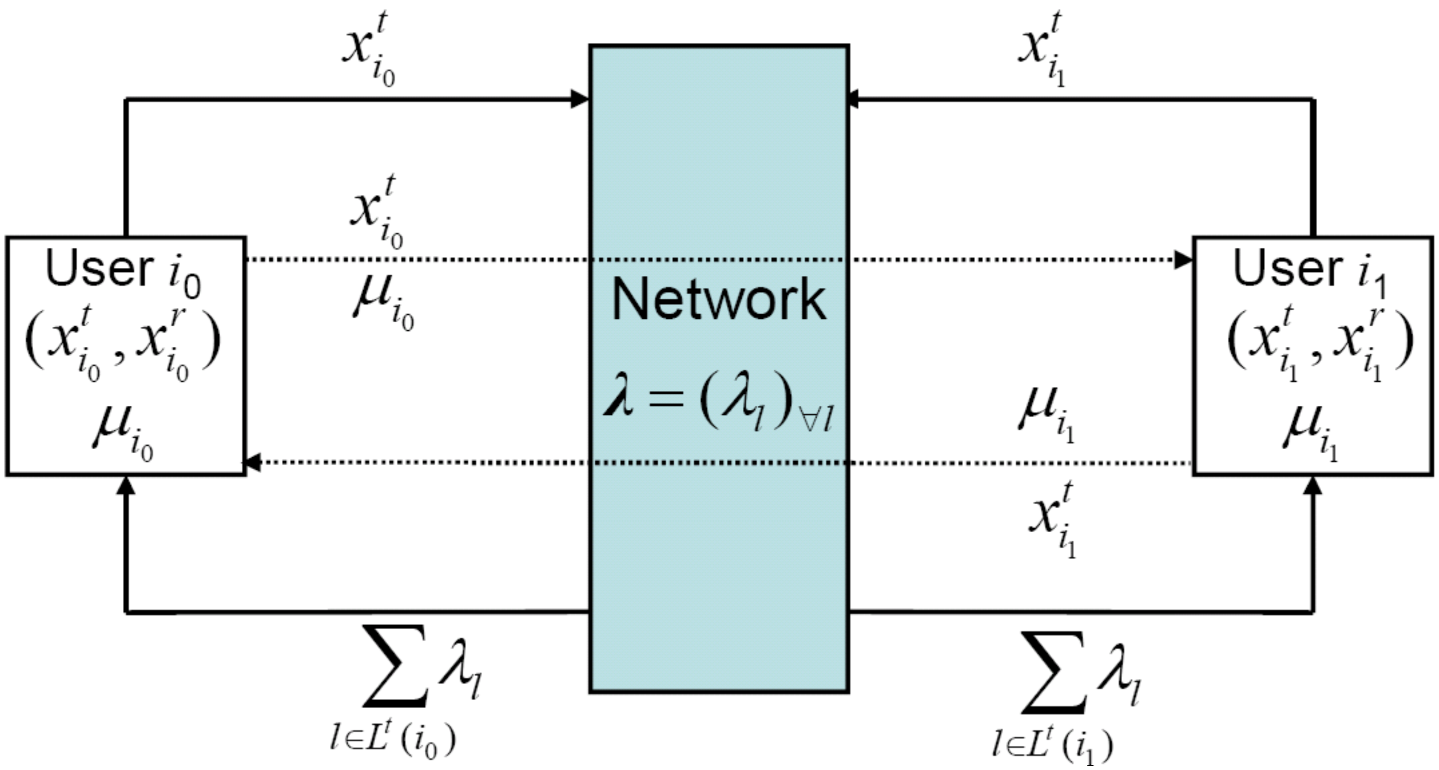
$$\mu_{i_0}^{(n+1)} = \mu_{i_0}^{(n)} - a^{(n)} \left(x_{i_1}^t(\boldsymbol{\lambda}^{(n)}, \boldsymbol{\mu}_0^{(n)}, \boldsymbol{\mu}_1^{(n)}) - x_{i_0}^r(\boldsymbol{\lambda}^{(n)}, \boldsymbol{\mu}_0^{(n)}, \boldsymbol{\mu}_1^{(n)}) \right), \ \forall i \in N$$

and

$$\mu_{i_1}^{(n+1)} = \mu_{i_1}^{(n)} - a^{(n)} \left(x_{i_0}^t(\boldsymbol{\lambda}^{(n)}, \boldsymbol{\mu}_0^{(n)}, \boldsymbol{\mu}_1^{(n)}) - x_{i_1}^r(\boldsymbol{\lambda}^{(n)}, \boldsymbol{\mu}_0^{(n)}, \boldsymbol{\mu}_1^{(n)}) \right), \ \forall i \in N,$$

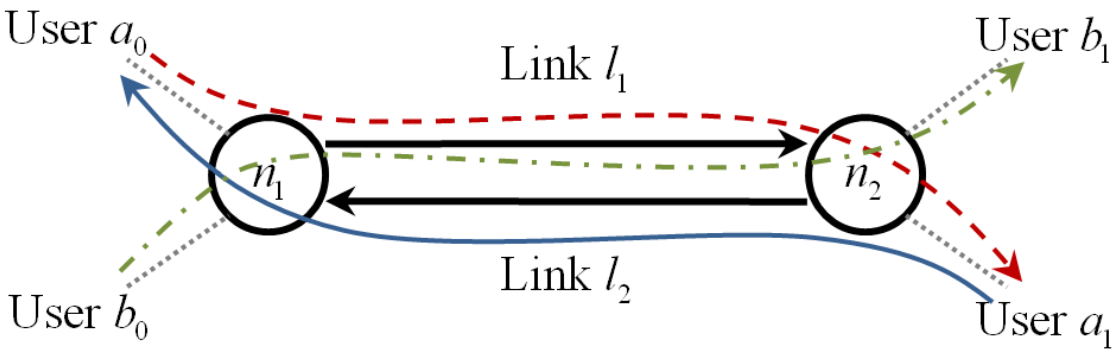
where $a^{(n)}$ is a step size at iteration n and $[c]^+ = \max\{c, 0\}$.

Diagram for distributed algorithm



Simulation

- **Simulation topology**



- **Capacity:** $C_{l_1} = 6, C_{l_2} = 2$
- **Utility function**

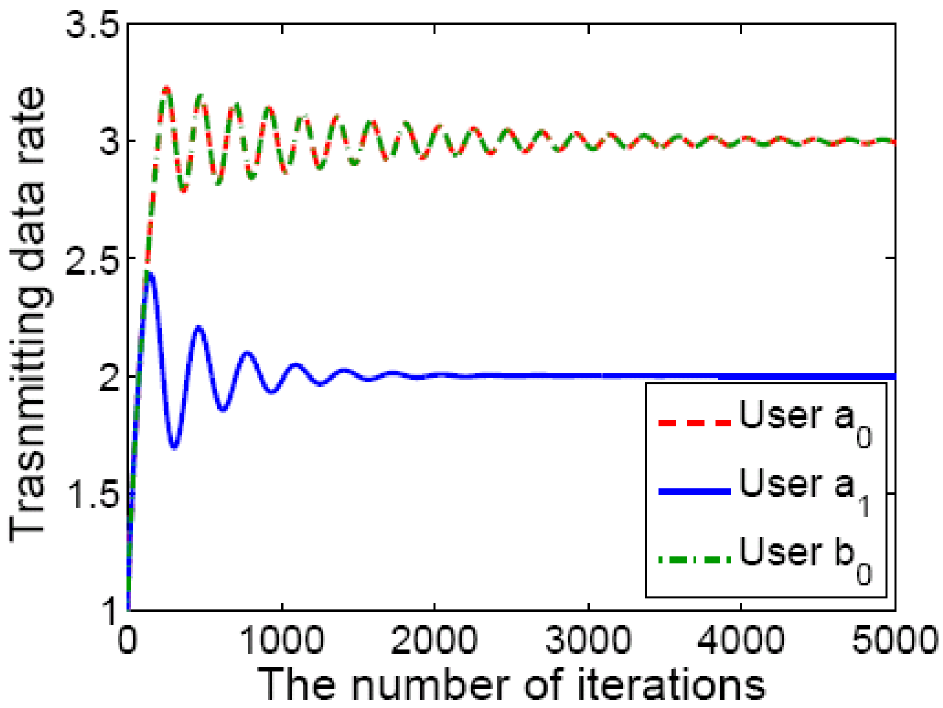
$$u_{i_j}^t(x_{i_j}^t) = \log x_{i_j}^t, \ u_{i_j}^r(x_{i_j}^r) = \log x_{i_j}^r$$

$$\begin{aligned} U_{i_0}\left(u_{i_0}^t\left(x_{i_0}^t\right), u_{i_0}^r\left(x_{i_0}^r\right)\right) &= \min\left[u_{i_j}^t(x_{i_j}^t), u_{i_j}^r(x_{i_j}^r)\right] \\ &= \min\left[\log x_{i_j}^t, \log x_{i_j}^r\right] \\ &= \log\left(\min\left[x_{i_j}^t, x_{i_j}^r\right]\right) \end{aligned}$$

Simulation

❖ **Previous flow control algorithm**

- Considering only transmitting session



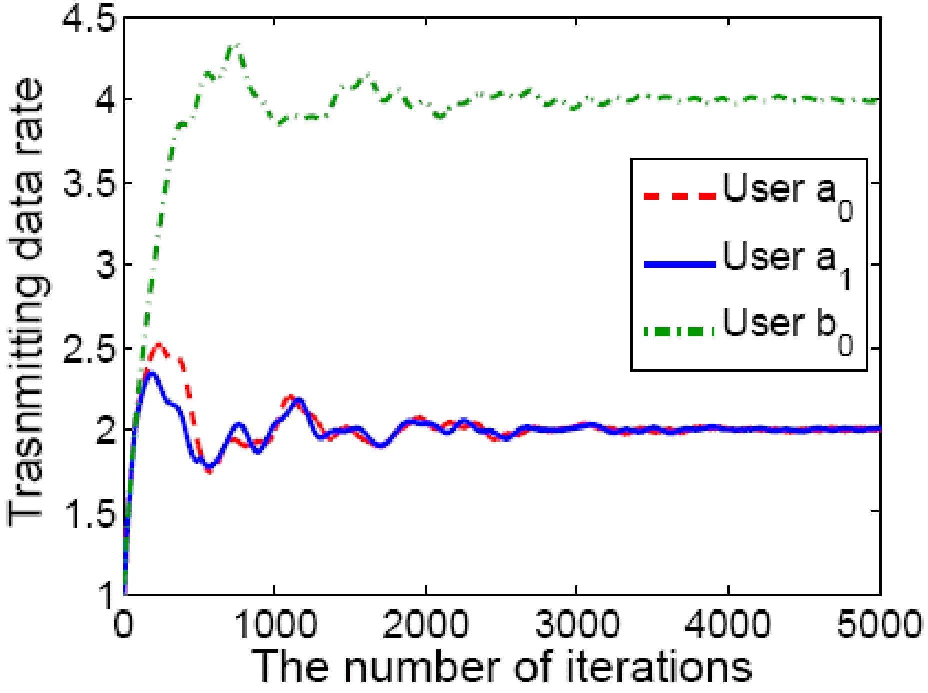
$$x_{a_0}^t = x_{b_0}^t = 3,\ x_{a_1}^t = 2$$

Network Utility : 3.58

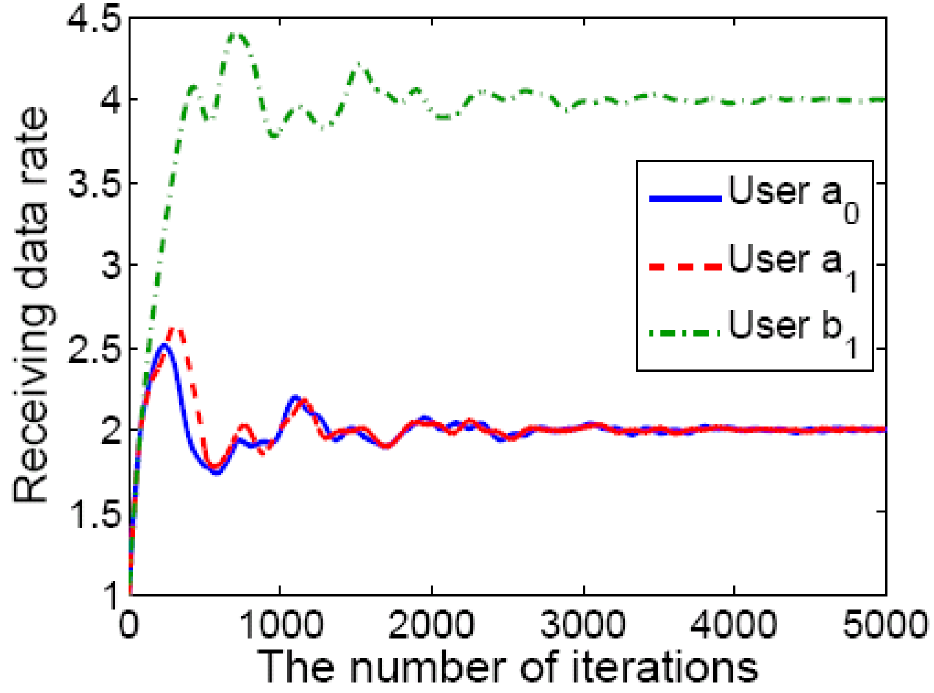
Simulation

❖ **Our flow control algorithm**

- Considering both transmitting and receiving sessions



(a) Transmitting data rates ($x^t_{a_0}$, $x^t_{a_1}$ and $x^t_{b_0}$).



(b) Receiving data rates ($x^r_{a_0}$, $x^r_{a_1}$ and $x^r_{b_1}$).

$$x^t_{a_0} = x^t_{a_1} = 2,\ x^t_{b_0} = 4$$

Network Utility : 4.16 (16% ↑)

Conclusion

- Unlike the previous approach in which user's satisfaction to the service is modeled considering only its transmitting session, we modeled it considering not only its transmitting session but also its receiving session through its user-level utility function.

- The results show that the algorithm developed with session-level utility functions may not provide the optimal flow control with respect to user-level utility functions, while our algorithms can provide the optimal flow control.